SOLVING A MULTI OBJECTIVE PROBLEM

Multi-Objective Optimization Problems

Most real-world decisions are, typically, made based on several conflicting performance criteria. As a field of operational research, multi-criteria decision-making (MCDM) is concerned with structuring and solving such problems. Multi-objective optimization (MOO)is an area of MCDM that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. In the single-objective optimization problem, the superiority of a solution over other solutions is easily determined by comparing their objective function values. Thus, in those problems, there exist either a single optimal solution or alternative optimal solutions giving the optimal objective function value. However, in MOO problems, there does not typically exist a feasible solution that optimizes all objective functions simultaneously. Therefore, attention is paid to Pareto optimal solutions; that is, solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives. Consider the following problem (P.1) formulated as a multi objective mathematical model

$$\text{Min } \{f_1(\vec{x}), f_2(\vec{x})\}$$
$$\textit{subject to}$$
$$3x_1 - 4x_2 \geq 3,$$
$$2x_1 - x_2 \leq 7,$$
$$x_1, x_2 \geq 0,$$

where $f_1(\vec{x}) = x_1 + x_2$, $f_2(\vec{x}) = -x_1$, and $\vec{x} = (x_1, x_2)$ is the vector of decision variables.

If P.1 is solved as a single objective problem by considering only $f_1(\vec{x})$, the optimal solution is (1, 0) and optimal $f_1(\vec{x})$ value is 1. The corresponding $f_2(\vec{x})$ value is -1.

If it is solved by only considering $f_2(\vec{x})$, the optimal solution is (5, 3) and optimal objective function value is -5. The corresponding $f_1(\vec{x})$ value is 5.

Weighted Sum Method

Procedure

The weighted sum method is a simple well-known method to solve multi objective problems. It optimizes a new optimization problem with a single objective function, which is a positively weighted convex sum of the objectives. In this method, a MOO problem with p objectives is formulated as follows:

$$\text{Min } \sum_{i=1}^{p} w_i f_i(\vec{x})$$
$$\textit{subject to}$$
$$\vec{x} \in \boldsymbol{X},$$

where $\boldsymbol{X}$ is the feasible region, p denotes the number objectives, w_i is the weight assigned to objective i, $i \in \{1, 2, \ldots, p\}$, $\sum_{i=1}^{p} w_i = 1$ and $w_i > 0$ $\forall i$.

Note that weights are predetermined parameters of the model. The problem can be solved for different weight assignments to find Pareto optimal solutions. Different weight assignments may not always yield a different Pareto optimal solution.

Case

The decision variables ($x1$ and $x2$) are real numbers. Write a macro in VBA reporting 4 Pareto optimal solutions using the weighted sum method when triggered by the button in the optimization sheet. While solving the optimization problems, use Excel Solver.

$$Max\ f1\ (\vec{x}) = x1 - x2$$
$$Max\ f2(\vec{x}) = x2$$
subject to
$$x1 + 2x2 \leq 12,$$
$$2x1 + x2 \leq 12,$$
$$x1 + x2 \leq 7,$$
$$x1 - x2 \leq 9,$$
$$-x1 + x2 \leq 9,$$
$$x1 + 2x2 \geq 0,$$
$$x1 - 3x2 \leq 4,$$
$$2x1 - x2 \leq 10,$$
$$x1,\ x2 \geq 0,$$